On Generalized Cooperative Systems and the Computation of their Solution Envelopes

D. de Pereda[1], S. Romero-Vivo[2], B. Ricarte[2], J. Bondia[1]

[1]Institut Universitari d'Automàtica i Informàtica Industrial, [2]Institut Universitari de Matemàtica Multidisciplinar, Universitat Politècnica de València
dpereda@upvnet.upv.es, sromero@imm.upv.es, bearibe@mat.upv.es, jbondia@isa.upv.es

Mathematical models have been traditionally used to mimic real-life situations in many diverse fields, such as biology, economics or human sciences. However, independently to the complexity of the model proposed, there is usually some mismatch between the model and the real phenomenon, yielding to non-modelled dynamics. Furthermore, a common characteristic of real processes is variability, leading to parametric uncertainty. For these reasons, the exact values of the initial conditions and the parameters of the model are unknown, but they can be bounded by intervals. When these values are assumed to be known, just a single solution is possible. However, if parametric uncertainty is considered, there is a set of possible solutions for the model. Clearly, this fact motivates the computation of tight solution envelopes.

A common approach on literature to compute guaranteed solution envelopes is to perform a monotonicity analysis of the parameters and initial conditions of the cooperative systems, that is, by taking the ordering induced by the positive orthant. In this event, it is known that if the monotonicity conditions are satisfied, it is possible to compute the exact solution envelopes. However, if these conditions are not satisfied, the solution envelopes may produce a significant overestimation with respect to the numerical simulations. Nevertheless, sometimes, for example for certain population models, it is worth considering a new class of systems named generalized cooperative systems, emerging from choosing any arbitrary cone as the order cone. In this work, the computation of tight solution envelopes for generalized cooperative systems under parametric uncertainty is addressed. Namely, a generalized cooperativity analysis is conducted in order to compute tight solution envelopes and in this way, to extend the previous results for cooperative systems.

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