Video analysis of the bouncing ball system *

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Abstract

The bouncing ball model is one of the simplest dynamical systems exhibiting a great variety of behaviors ranging from periodic to chaotic motion [1]. With the increase of computational capabilities, video analysis is a technique of increasing interest to analyze the movement of the objects in a scene.

In this work, we record the motion of a ball bouncing on a platform, process the images to obtain the optical flow and compute the spectrum of the velocities to assess the behavior of the system.

The classical Lucas-Kanade's method allows tracking the trajectory of point along the video sequence [2]. Following a point of the ball, we are able to determine the characteristics of its motion, in particular, the position and time of its impacts with the platform. The Farnebäk's method computes the optical flow of a region of the frame [3]. The spectrum of the average flow is used to find the frequencies present in the movement of the ball.

We also adjust the parameters of a mathematical model of the system to perform a computer simulation and compare the results with the experimental ones.

In our experiment, we record a table tennis ball that bounces on the membrane of a loudspeaker. The loudspeaker is driven by a sinusoidal voltage with controlled amplitude. For different amplitudes, we have studied different states of the system.

It is interesting to observe that there are different possible states of the bouncing ball system for a given value of the control parameters. At low loudspeaker amplitudes, the ball can either rest in permanent contact with the membrane or bounce at the same frequency as the membrane vibration.

Increasing the amplitude, the system suffers a bifurcation and the ball performs two bounces of different height in two periods, or higher equal bounces spanning two periods each.

We have also recorded the ball performing two different bounces in four periods or irregular bounces at even higher loudspeaker amplitudes.

In each case, we obtain the spectrum of the movement from the optical flow results and compare the tracked trajectory with the corresponding simulated trajectory. We find a good agreement between the experimental results and the simulations, which confirms the quality of the experimental setup and the suitability of the dynamical model.

Keywords: Bouncing ball model, video analysis, dynamical systems, optical flow, period coupling bifurcation, chaos.

References

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