Computing Survival Functions of the Sum of Two Independent Markov Processes. An Application to Bladder Carcinoma Treatment.

B. García-Mora*, C. Santamaría*, G. Rubio*, J.L. Pontones**

(*) Instituto Universitario de Matemática Multidisciplinar. Universitat Politècnica de València. Spain.
<magarmo5> <crisanna> <grubio>@mat.upv.es

(**) Hospital Universitario La Fe. Valencia. Spain.
pontones_jos@gva.es

The aim of this study is to calculate the survival function associated to bladder carcinoma considering two absorbing (terminating) events. For this purpose we consider the study of the sum of two independent phase–type (PH) distributed variables, each of them being associated with a Markovian process with one absorbing state. We calculate the distribution function of the sum of these two variables resulting a new PH–distributed function of higher order. As the order increases in the new function the exponential function of a block upper triangular matrix is calculated in terms of its respective blocks to reduce the dimension of the problem. For this we make use of the Fréchet derivative, the Kronecker matrix form as well as the Kronecker product and the exterior sum. We then study the Markov process associated with the sum variable and this new distribution function. In a second step and in order to address potencial computational problems two approximations to the distribution function are calculated. Finally, all the obtained results are applied to a numerical example of bladder carcinoma data in survival analysis.

Key words: Markov process, Fréchet derivative, Kronecker product, Phase-type distribution, Survival analysis, Bladder carcinoma.