Modelling Metamaterial Acoustics on Spacetime Manifolds

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Abstract

Transformation acoustics focuses on the design of advanced acoustic devices by employing sophisticated mathematical transformation techniques for the engineering of acoustic metamaterials—materials mostly artificially fabricated with acoustic properties beyond those encountered in Nature [1–3]. Industrial applications in this field have wide repercussions and range from acoustically improving concert halls to constructing ships and submarines invisible to sonar detection.

We introduce differential-geometric methods together with a variational principle and show how they form the basis for a powerful framework with the aim to control acoustic waves in these industrial applications. As a main requirement, the acoustic wave equation is invariant under certain coordinate transformations. In this formalism we define a scalar acoustic Lagrangian function on a smooth spacetime manifold, eventually shown to produce the general constitutive relations between the acoustic parameters of the metamaterial under consideration (bulk modulus and mass-density tensor).

Previously, we have elaborated a Lagrangian framework to describe macroscopic electrodynamics for optical metamaterials [4] and diffusion on curved manifolds [5]. Here, we use a similar approach to derive the equations of motion for non-dissipative acoustic phenomena from a fundamental Lagrangian density. In electrodynamics Maxwell's equations are already inherently covariant, acoustics however does not possess this advantage. Nevertheless, we succeed in reformulating Hamilton's principle for acoustics in a fully covariant fashion for spacetime [6].

Finally, we conclude with a practical example of this theory to demonstrate how the design of advanced acoustic devices heavily relies on the tuning of the material parameters of the physical and virtual spaces in order to implement a desired spacetime geometry.

References

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