Discontinuous Galerkin method for numerical solution of exotic option pricing model [1]

Jiří Hozman

Department of Mathematics and Didactics of Mathematics,
Faculty of Science, Humanities and Education,
Technical University of Liberec,
Studentská 2, 461 17 Liberec, Czech Republic,
e-mail: jiri.hozman@tul.cz

ABSTRACT

During the last decade, financial models have acquired increasing popularity in option pricing. The valuation of different types of option contracts is very important in modern financial theory and practice – vanilla and especially exotic options have become very popular speculation instruments in recent years. The problem of determining the fair price of such an option is standardly formulated in well-known *Black-Scholes* equation, see [1].

At the same time a huge amount of literature has been devoted to the solving of this equation or its modification. The performance demands on the valuation process are very high in this case. Moreover, closed forms of the solution to derivative pricing problems are not often easily computable and analytic solutions cannot be constructed in some models generally. Therefore, numerical methods for option pricing have been analyzed by many authors and several techniques have been developed to obtain efficient pricing algorithms, see, e.g. [4], and the references cited therein.

The demand for performance has led some to use approximations with the aid of *finite difference* (FD) method or *finite element* (FE) method, see [2]. These methods have also its limitations in the treatment of exotic options, which have special features that need to be taken into account when pricing them. Such as in the case of the discrete barrier option, where it is more suitable to construct discontinuous solutions with respect to time and space coordinates.

Therefore, in this paper, we focus on *discontinuous Galerkin* (DG) methods, which combine the advantages of FE method together with discontinuous approach, for survey see [3]. This method is based on piecewise polynomial, generally discontinuous approximations. From this point of view, DG methods seem to be very promising tool for the numerical simulation of option pricing and provides *robust* and *high-order accurate* approximations of solutions resulting from the Black-Scholes equation. Moreover, the DG concept can be easily extended by hp-adaption techniques, which enable better resolving of occurred special properties of certain types of options with respect to computational meshes as well as polynomial approximation degrees. Finally, one of the most important advantages in practice common for FE and DG approach is that sensitivity measures, such as the *greeks*, can be calculated more exactly than using FD methods.

In order to investigate the properties of proposed DG approach we carry out some numerical experiments for pricing barrier options illustrating the potency of this DG scheme.

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